Analysis on aggregation and block smoothers in multigrid methods for block Toeplitz linear systems *

Matthias Bolten[1], Marco Donatelli[2], Paola Ferrari[3], Isabella Furci[4]

[1]School of Mathematics and Natural Sciences, University of Wuppertal, Wuppertal, Germany.
email: bolten@uni-wuppertal.de

[2]Department of Science and High Technology. University of Insubria, Como, Italy.
email: marco.donatelli@uninsubria.it

[3]Department of Mathematics, University of Genoa, Genoa, Italy
School of Mathematics and Natural Sciences, University of Wuppertal, Wuppertal, Germany.
email: paola.ferrari@dima.unige.it, email: ferrari@uni-wuppertal.de

[4]Department of Mathematics, University of Genoa, Genoa, Italy.
email: isabella.furci@dima.unige.it

Abstract

We present novel improvements in the context of symbol-based multigrid procedures for solving large block Toeplitz linear systems [4]. We study the application of aggregation-based grid transfer operators that transform the symbol of a block Toeplitz matrix from matrix-valued to scalar-valued at the coarser level [1]. The convergence analysis of the TGM reveals the connection between the features of the scalar-valued symbol at the coarser level and the properties of the original matrix-valued one [3]. This permits to prove the convergence of a V-cycle multigrid involving classical grid transfer operators for scalar Toeplitz systems at the coarser levels [2].

Moreover, we extend the class of suitable smoothers focusing on the efficiency of block strategies, particularly the relaxed block Jacobi method. General conditions on smoothing parameters are derived, with emphasis on practical applications where these parameters can be calculated with negligible computational cost.

We test the proposed strategies on linear systems stemming from discretization of differential problems. The numerical results show computational advantages compared to existing methods for block structured linear systems.

References

[1] C. An and Y. Su. *An Aggregation-Based Two-Grid Method for Multilevel Block Toeplitz Linear Systems*. J. Sci. Comput., **98** (2024), no.3, Paper No. 54, 47 pp.

[2] A. Aricò and M. Donatelli. *A V-cycle multigrid for multilevel matrix algebras: proof of optimality*. Numer. Math. **105** (2007), no.4, 511–547.

[3] M. Bolten, M. Donatelli, P. Ferrari, and I. Furci. *A symbol-based analysis for multigrid methods for block-circulant and block-Toeplitz systems*. SIAM J. Matrix Anal. Appl.**43** (2022), no.1, 405–438.

[4] M. Bolten, M. Donatelli, P. Ferrari, and I. Furci. *Analysis on aggregation and block smoothers in multigrid methods for block Toeplitz linear systems*. arXiv:2403.02139v1. (submitted)

*The work is partially supported by INdAM - GNCS Project "Analisi e applicazioni di matrici strutturate (a blocchi)" CUP E53C23001670001 and by MUR – PRIN 2022, grant number 2022ANC8HL. The work of Isabella Furci is supported by #NEXTGENERATIONEU (NGEU) and funded by the Ministry of University and Research (MUR), National Recovery and Resilience Plan (NRRP), project MNESYS (PE0000006) – A Multiscale integrated approach to the study of the nervous system in health and disease (DN. 1553 11.10.2022).