Mathematical imaging and signal processing

Session at the *6th Dolomites Workshop on Constructive Approximation and Applications 2024, by W. Erb and I. M. Bulai*

Last update: June 28, 2024

List of contributions

Estimating cortical brain functional network from M/EEG data by means of ℓ_1 optimization.

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Abstract:

During the past years, studying brain functional connectivity (FC) from magneto/electro - encephalographic (MEEG) data has become a popular method to explore how brain networks behave in both healthy and diseased conditions, expanding the field of human brain mapping. When dealing with MEEG data, functional connectivity is usually estimated with a two-step approach that involves: (i) solving an inverse problem for the reconstruction of brain activity (ii) computing proper metrics, such as the cross-power spectrum, for evaluating the statistical dependencies between different brain areas.

From a mathematical view-point, the MEEG inverse problem can be written as $y(t) = Gx(t) + n(t)$, where $y(t) \in \mathbb{R}^M$, $x(t) \in \mathbb{R}^N$, and $n(t) \in \mathbb{R}^M$ are realisations of multivariate stochastic processes representing MEEG recordings, brain activity, and measurement noise, respectively, and $G \in \mathbb{R}^{M\times N}$ is the leadfield matrix or forward operator. In this setting, a regularized estimate $x_\lambda(t)$ of the neural activity $x(t)$ is obtained from a minimization problem with Tikhonov regularization. After estimating the brain activity $x_\lambda(t)$, the cross-power spectrum $S^{x_\lambda}(f)$ of $x_\lambda(t)$ is computed as the Fuorier transform of the correlation function between $x_{\lambda(t)bda}$ and itself.

This two-step approach has been demonstrated to be in fact suboptimal causing the detection of many spourious connection inte the final connectivity estimate.

We offer here a one-step approach for functional connectivity estimation. In particular, from $y(t) = Gx(t) + n(t)$, it follows that the cross-power spectrum of $y(t)$ is

$$S^y(f) = GS^x(f)G^T + S^n(f) \tag{1}$$

and that the l_1-regularized solution is computed by solving

$$\mathcal{S}^x_\lambda(f) = \arg\min_{\mathcal{S}^x}\{\|\mathcal{G}\mathcal{S}^x(f) - \mathcal{S}^y(f)\|_2^2 + \lambda\|\mathcal{S}^x(f)\|_1^2\}, \tag{2}$$

where $\mathcal{S}^y$ and $\mathcal{S}^x$ are obtained by vectorizing S^y and S^x, respectively, and $\mathcal{G}$ is given by

$$\mathcal{G} = \begin{pmatrix} G\otimes G & 0 \\ 0 & G\otimes G \end{pmatrix}.$$

In particular, we exploit the tensorial structure and properties of $\mathcal{G}$ to present an efficient strategy for the computation of $\mathcal{G}\mathcal{S}^x(f)$, making the algorithm feasible and reduced computational cost and time.

We will simulate data to show some preliminary results on the effectiveness of the proposed approach and on its advantages with respect to the classical two–step approach, especially focusing on the smaller number of spourious connection that can be achieved by the one-step approach.

References

Joint work with: Isabella Furci (University of Genoa), Sara Sommariva (University of Genoa), Michele Piana (University of Genoa, IRCCS San Martino Hospital).

Acknowledgments: